STOCHASTIC SENSITIVITY ANALYSIS FOR FINANCE

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ABSTRACT

In a risk management of derivative securities, sensitivities of an option price are an important measure of the risk and there exists a great need for their efficient computation. Commonly referred to as the Greeks in finance, e.g., *Delta, Gamma, Vega,* etc., they are mathematically defined as the partial differential sensitivity coefficients of the option price with respect to the underlying model parameters of the securities that are typically described by well-known Black-Scholes equation. When there are no closed formulas for option prices, one has to use numerical methods to approximate these prices and their Greeks (sensitivities).

In financial engineering, finite difference approximations are heavily used to simulate the Greeks by means of Monte Carlo or Quasi Monte Carlo procedures. However, the finite difference approximation soon becomes inefficient particularly when payoff functions are complex and discontinuous. This is often the case when we deal with exotic options such as American options, Asian options, lookback options, etc. To overcome this difficulty, Fournie et al. (1999) (2001) suggested the use of Malliavin calculus, by means of an integration by parts, to shift the differential operator from the payoff function to the underlying diffusion (e.g., Gaussian) kernel, introducing a weighting function. In this paper, we extend the Malliavin calculus approach, and describe a constructive method for a stochastic sensitivity analysis in financial engineering. The present approach enables the computation of the Greeks without resorting to direct differentiation of the payoff function.

We especially focus our attention to numerical computations of Greeks through Monte Carlo methods, finite difference approximations, and Malliavin calculus approaches. Based on intensive numerical simulations, some variance reduction techniques will be discussed.

References

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